Electric current consists of motion of charge from one region to another. (I hope you remember that electrostatics was the study of charges at rest) It is a scalar quantity.

If the charges follow a conducting path that forms a closed loop, the path is called an electric circuit.

If ΔQ is the amount of charge that passes through an area in time Δt, then the average current

$$I_{avg} = \frac{\Delta Q}{\Delta t}$$

Current can be steady or variable.

If $\frac{\Delta Q}{\Delta t}$ is constant over any time interval taken, the current is steady or else variable. When the current is variable we refer to instantaneous value of current. $I = \ell im_{\Delta t \to 0}\left(\frac{\Delta Q}{\Delta t}\right) = \frac{dQ}{dt}$

S.I unit of current is ampere. (A) One ampere is equivalent to one coulomb of charge passing through the cross section of a conductor in one second. (I hope you remember, coulomb is the unit of charge)

Current density: If you consider a small area Δs around a point p in a conductor, perpendicular to the flow of charges and the current through the area is Δi, then the current density is $j_{avg} = \frac{\Delta i}{\Delta s}$

If you want the current density exactly at the point p, then you have to make the area around the point p that you have taken as small as possible. (Like you made the time interval Δt small while defining the instantaneous value of the current) This mathematically means $j = \ell im_{\Delta s \to 0}\frac{\Delta i}{\Delta s} = \frac{di}{ds}$ it is a vector quantity. Its direction is same as that of the current. Remember that the current direction conventionally is same as the direction of flow of positive charges. (Opposite to the direction of flow of negative charges) $di = \vec{j} \bullet d\vec{s}$

If the current I is distributed uniformly over the area S, and is perpendicular to the area, current density is $j = \frac{I}{S}$

If the area vector (taken perpendicular to the area) makes an angle θ with the direction of the current, then $j = \frac{I}{S\cos\theta}$ where the denominator is the component of the area taken in the direction of the current.

Remember that the electrons, which are enough in number in a metal, are continuously in random motion. This motion does not constitute any current because it is random. If you find 100 electrons going towards right of you, you will find an equal number going towards your left making the net charge movement zero.

When a potential difference is applied across the ends of the conductor, an electric field is created inside the conductor. (Remember that when you had charged a conductor as in electrostatics, electric field was not inside the conductor but was only outside it) The electrons experience a force due to the electric field and start moving opposite to the direction of the field systematically. This is called drifting. The random motion which was associated with them earlier is still there but now they have an additional motion which is systematic. They collide often and lose part of their kinetic energy which is the cause for increase in vibrational energy and also rise in temperature.

The force experienced by the electron due to the field $\vec{E}$ is $\vec{F} = -e\vec{E}$

Can you understand why it is minus? It is opposite to the field $\vec{E}$ as the particle experiencing the force is electron which is negatively charged. Hence acceleration will be $\vec{a} = \frac{-e\vec{E}}{m}$ where m is the mass of the electron.

The drift velocity is the average velocity of all the electrons. (Remember this average in the absence of potential difference and electric field was zero earlier) Consider any i[th] electron, its velocity is $\vec{v}_i = \vec{u}_i + \vec{a}t_i$ where t_i is the time the particular electron spends between the collisions. $\vec{v}_d = \frac{\sum \vec{v}_i}{N}$ Where N is the total number of electrons in the conductor across which potential difference is applied.

Hence $\vec{v}_d = \frac{\sum \vec{u}_i}{N} + \frac{\sum \vec{a}t_i}{N} = \frac{\sum \vec{u}_i}{N} + \vec{a}\frac{\sum t_i}{N}$

Now $\sum \vec{u}_i$ = 0 as every electron after every collision starts afresh in an arbitrary direction. So, average taken over all electrons turns to be zero. $\frac{\sum t_i}{N} = \tau$ Which is called relaxation time.

Hence $\vec{v}_d = \vec{a}\tau = \frac{-e\vec{E}}{m}\tau$ (again minus only indicates that the drifting is opposite to the field)

Expression for current and current density in terms of drift speed:

If L is the length of the conductor and $\vec{v}_d$ is the drift speed, time taken by the electrons to cross the conductor's length is $t = \frac{L}{v_d}$

If the number of electrons present per unit volume of the conductor is n, their total number = nAL

Total charge in the volume of the conductor is $q = (nAL)e$

Current is $I = \frac{q}{t} = \frac{(nAL)e}{t} = \frac{(nAL)e}{\left(L/v_d\right)} = nAev_d$

But current density is $j = \frac{I}{A} = nev_d$ or $\vec{j} = ne\vec{v}_d$

Derivation of Ohm's law and expression for resistance:

If a potential difference V is applied across a conductor of length L, electric field inside the conductor E = V/L (Remember $\vec{E} = -\frac{dV}{d\vec{r}}$)

But $\vec{v}_d = \frac{-e\vec{E}}{m}\tau$

Therefore $\vec{v}_d = \frac{-eV}{Lm}\tau$ ------------ (1) (by substituting the value of E)

Also $I = nAev_d$ which implies $v_d = \frac{I}{nAe}$ ------------ (2)

Therefore $\frac{I}{nAe} = \frac{eV}{Lm}\tau$ from (1) and (2)

Hence $V = I\left(\frac{Lm}{nAe^2\tau}\right)$ The term in the bracket is a constant for a given conductor of length L and area of cross section A.

This shows V is proportional to I and the proportionality constant is called the resistance of the conductor. This is the mathematical form of Ohm's law. ($V = IR$)

Resistance can be written as $R = \left(\frac{m}{ne^2\tau}\right)\frac{L}{A} = \rho\frac{L}{A}$

Where $\rho = \left(\frac{m}{ne^2\tau}\right)$ is called as the resistivity of the material of the conductor.

You can see that resistivity depends on number density n and relaxation time τ . For conductors n is large and hence resistivity is small. As temperature increases, relaxation time decreases. So, resistivity of a conductor depends on temperature. Hence the definition of Ohm's law, "temperature remaining constant, current I varies directly as the potential difference applied across its ends if physical conditions (A and L) are kept constant.