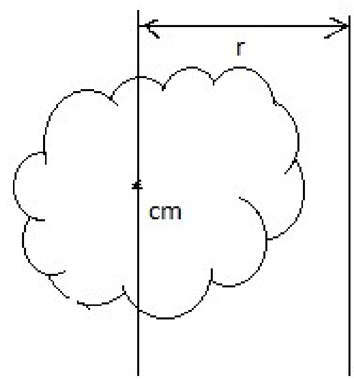
Some interesting conceptual questions on rotational motion.

Synopsis:

Moment of Inertia: This is important in the study of system of particles that are rotating. The role of M.I in the study of rotational motion is same as mass in the study of translatory motion. M.I gives a measurement of resistance of the body to a change in its rotational inertia. If the body is at rest, the larger the M.I of the body, more difficult it is to put it into rotational motion. Similarly if the body is already rotating, greater the M.I more difficult it is to stop its rotational motion. This depends on the mass as well as its distribution around the axis.

For a particle of mass m about an axis at a distance r from it $I = mr^2$

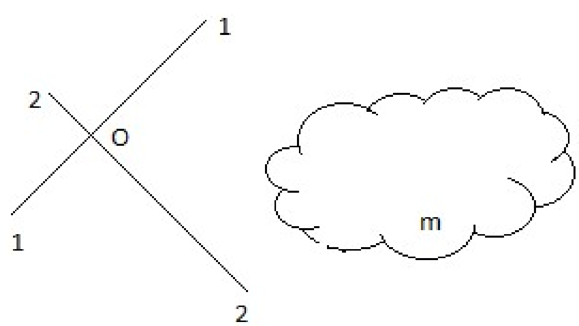
Theorem's on M.I:



Parallel axis theorem: $I = I_{cm} + mr^2$

This is applicable for any rigid body, two dimensional or three dimensional.

Perpendicular axis theorem: It is applicable only for a plane lamina, that is a two dimensional body.



If the M.I of the plane lamina is known about axes 1-1 and 2-2 which are perpendicular to each other and in the plane of the lamina, their sum that is $I_1 + I_2$ gives the M.I of the lamina about a third axis passing through the intersection of those two and perpendicular to them that is I_O.

Torque: It is $\vec{r} \times \vec{F}$ where $\vec{r}$ is the position vector of the point of application of the force $\vec{F}$ with respect to some point about which the torque is being written. When a rigid body is rotating about a fixed axis and a force is applied on it at some point then we are concerned with the component of the toque of this force about the axis of rotation and the total torque. This

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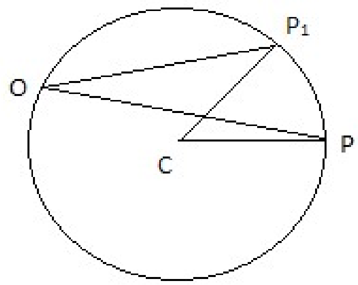
component about the axis of rotation is independent of the choice of the origin as long as it is chosen on the axis.

$\sum \vec{\tau} = I\vec{\alpha}$ Is valid only for rigid bodies. This does not hold for a non-inertial frame of reference. However there exists a very special case when the equation does hold good even if angular acceleration is measured from a non-inertial frame. The case is when the axis of rotation passes through the centre of mass.

Angular velocity: of a particle about a point is $\omega = \frac{d\theta}{dt} = \frac{v_{\perp}}{r}$ it is also the component of the velocity perpendicular to $\vec{r}$ divide by the distance of the point P from O at that instant.

$\vec{v} = \vec{\omega} \times \vec{r}$



If a particle P is moving in a circle, its angular velocity about the centre of the circle C that is ω_c is 2 times the angular velocity about any point on the circumference of the circle that is ω_O.

Angular momentum: The angular momentum of a particle about a line is the component of the angular momentum of the particle about any point on the line. This is independent of the choice of the origin as long as the point is on the line.

For the relation $\vec{L} = I\vec{\omega}$ to be correct both the vectors should point in the same direction. This can happen only when the body is symmetric about the axis of rotation. By symmetry we mean for every mass element in the body there must be an identical mass element diametrically opposite and at the same distance from the axis of rotation. However $L = I\omega$ holds for every rigid body symmetric or not rotating about a fixed axis.

Angular momentum of a rigid body in combined rotation and translation: If O is a fixed point in an inertial frame, $\vec{L} = \vec{L}_{cm} + m\left(\vec{r}_{cm} \times \vec{v}_{cm}\right)$

$\vec{\tau} = \frac{d\vec{L}}{dt}$ Holds for a particle only if both the vectors are measured with respect to any point fixed in an inertial frame or the centre of mass of the system for a system of particles.

1. Can you find an axis of rotation about which the moment of inertia of the body say I_1 is smaller than its moment of inertia about an axis parallel to it and passing through its centre of mass say I_g?
2. Suppose you want to design a wheel and bring it down the hill without delivering any power to it and want the coasting speed to be maximum. How do you design? You want their moment of

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inertia about their centroidal axis to be minimum or maximum? Or does it not matter at all? Also can you think of achieving the same by keeping their moment of inertia same but changing their masses? If so, whether you would increase their masses or decrease? Assume no loss of energy.

3. Assume a particle to be in uniform circular motion and you are writing its angular momentum about the centre of the circle. Which do you think will remain constant about the angular momentum? Magnitude or direction? What is your answer if the particle were to be in a non-uniform circular motion?

4. If the torque acting on the particle and its angular momentum are in the same direction, does its angular momentum change in magnitude or direction or both or neither?

5. When a solid sphere rolls down without slipping what force produces a torque that causes angular acceleration about an axis passing through the centre of mass? What force produces a torque that causes angular acceleration about an axis passing through the point of contact with the surface?

6. A horizontal force F = mg/3 is applied on the upper surface of a uniform cube of mass m and side a which is resting on a rough horizontal surface having μ_s = 1/2. The distance between the lines of action of weight and normal reaction of the surface is

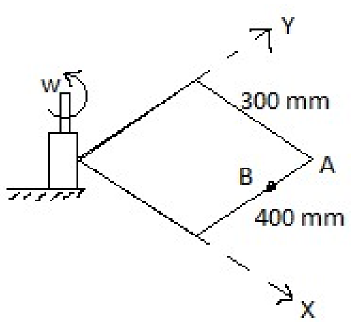
a) a/2 b) a/3 c) a/4 d) zero

7. A uniform disc of mass m rolls down an inclined plane with an acceleration a. The frictional force on the disc due to the surface is

a) 2ma b) (3/2) ma c) ma d) (1/2) ma

8. A uniform sphere of radius R is placed on a rough horizontal surface and given a linear velocity v towards your right and angular velocity ω in the anticlockwise direction as seen by you. The sphere comes to rest after moving some distance towards right. It follows that

a) v = ωR b) 2v = 5ωR c) 5v = 2ωR d) 2v = ωR



1. The rectangular plate is rotating about an axis passing through its corner and perpendicular to its plane with angular velocity w = 10 rad/s.

A. The magnitude and direction of the velocity V of the corner A is------ (5 m/s^2) ----- ($-4\hat{i}+3\hat{j}$)

B. The magnitude and direction of the acceleration of the corner A is----- (50 m/s^2) ---- ($-30\hat{i}-40\hat{j}$)

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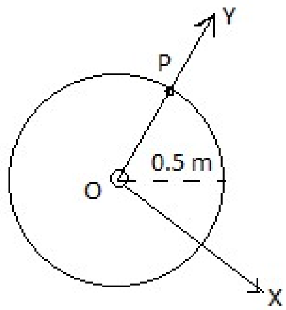
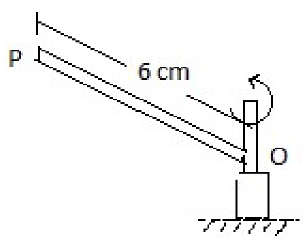
C. If the plate starts from rest and reaches an angular speed of 300 rad/s in 2 sec with a constant angular acceleration and the point B has an initial acceleration of 5.5 m/s^2 what is the distance of B from A? ----- (180.6 mm)



2. The circular disc rotates about its centre O in the clockwise direction. At a certain instant the point P on the rim has an acceleration $\vec{a} = -3\hat{i} - 4\hat{j}$ m/s^2. For that instant

A. The angular velocity w is ----- (√8 rad/sec)

B. The angular acceleration α is ---- (6 rad/sec^2)

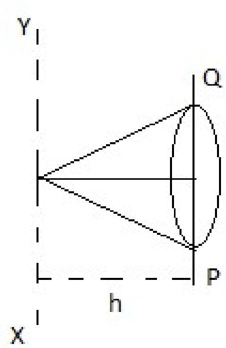


3. The rod starts from rest and acquires a speed of 60 rev/min in 2 sec. by rotating in the horizontal plane about the vertical axis shown with a constant angular acceleration.

A. The angular acceleration is ------ (10π rad/sec^2)

B. The time after which the acceleration vector of P makes an angle of 45^0 with the rod is --- (0.18 s)

C. The magnitude of acceleration of P at time t is ----- ($1.5\pi\sqrt{2}$ m/s^2)



4. The diagram shows a uniform solid cone of height h and mass m. If the M.I of the cone about X-Y is I, its M.I about PQ is

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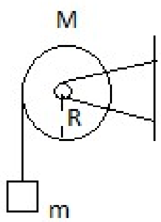
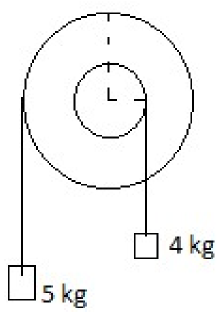
a) $I + Mh^2$ b) $mh^2 - I$ c) $I + 0.5mh^2$ *d) $I - 0.5mh^2$



5. A uniform disc of radius R and mass M is free to rotate about a fixed axis perpendicular to its plane and passing through its centre. A string is wrapped over its rim and a block of mass m is attached to the free end of the string as shown. The block is released from rest. If the string does not slip on the rim, find the acceleration of the block. Neglect the mass of the string.

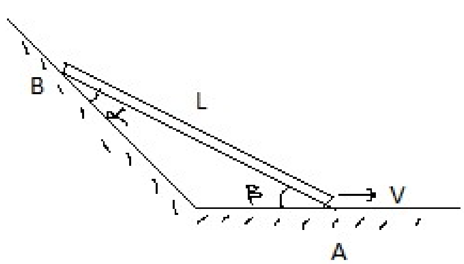
Ans: $\frac{2mg}{2m+M}$

6. Two forces $\vec{F}_1 = 2\hat{i} - 5\hat{j} - 6\hat{k}$ and $\vec{F}_2 = -\hat{i} + 2\hat{j} - \hat{k}$ are acting on a body at points (1, 1, 0) and (0, 1, 2) respectively. Find the torque acting on the body about the point (-1, 0, 1). Can you add the two forces first to get the resultant force and then write the torque due to the resultant force? Ans: $-14\hat{i} + 10\hat{j} - 9\hat{k}$



7. The M.I of the pulley system is 4 kgm^2. The radii of bigger and smaller pulleys are 2 m and 1 m respectively. The angular acceleration of the system is (in rad/sec^2)

*a) 2.1 b) 4.2 c) 1.2 d) 0.6



8. A rod of length L slides down along the inclined wall as shown. At the instant shown in the figure, the speed of the end A is v. What is the speed of B?

a) $\frac{v\sin\beta}{\sin\alpha}$ b) $\frac{v\sin\alpha}{\sin\beta}$ c) $\frac{v\cos\alpha}{\cos\beta}$ *d) $\frac{v\cos\beta}{\cos\alpha}$

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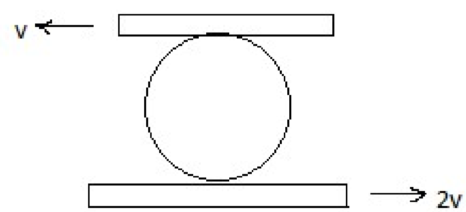
9. A particle moves in a circle with constant angular velocity ω about a point p on its circumference. The angular velocity of the particle about the centre C of the circle is

a) 2ω b) ω/2 c) ω d) none

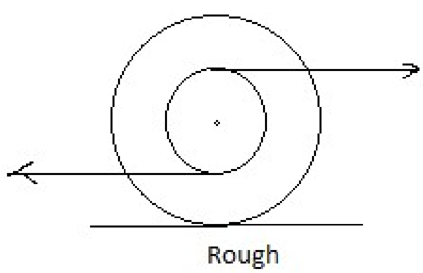
10. A particle of mass m is projected with a velocity V at an angle θ with the horizontal. Find its angular momentum about its point of projection when it is at the highest point of its trajectory.

Ans: $\frac{mv^3 \sin^2\theta \cos\theta}{2g}$

11. A sphere rolls without slipping on a rough horizontal surface with centre of mass having a constant speed v. The mass of the sphere is m and its radius is R. Find the angular momentum of the sphere about the point of contact. What is the kinetic energy of the sphere?



12. The disc of radius R is confined to roll without slipping while in contact with the two surfaces moving with velocities shown. What is the angular velocity of the disc?



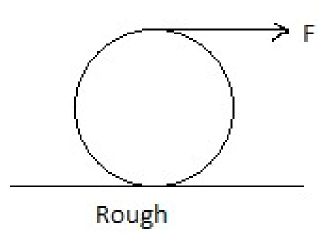
13. A spool is pulled horizontally by two equal and opposite forces as shown. Which of the following statements is correct?

a) The centre of mass moves towards the left

b) The centre of mass moves towards the right

c) The centre of mass remains at rest.

d) The net torque about the centre of mass of the spool is zero.



14. A ring of mass m and radius R is kept on a rough surface with a horizontal force F applied as shown. It rolls without slipping on the surface. Find the work done by the force F when the centre of mass moves a distance S.

a) FS b) 2 FS c) Zero d) 3FS/2

15. A boy is standing on a platform which is free to rotate about its axis. The boy holds an open umbrella in his hand. The axis of the umbrella coincides with that of the platform. The M.I of boy + platform is $3(10^{-3})$ kg-m^2 and that of the umbrella is $2(10^{-3})$ kg-m^2. The boy starts spinning the umbrella about its axis at an angular speed of 2 rev/sec with respect to himself. Find the angular velocity imparted to the platform. (0.8 rev/s)

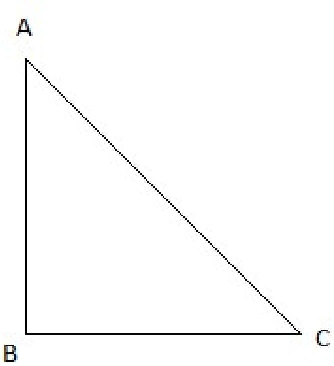
16. A wheel of M.I 0.1 kg-m^2 is rotating about a shaft at an angular velocity of 160 rev/min. A second wheel is set into rotation at an angular velocity of 300 rev/min and is coupled to the same shaft so that both the wheels finally rotate with a common angular speed of 200 rev/min. Find the M.I of the second wheel. (0.04 kg-m^2)

17. A kid of mass M stands at the edge of a platform of radius R which can freely rotate about its axis. The M.I of the platform is I. The system is at rest when a friend throws a ball of mass m and the kid catches it. If the velocity of the ball is v horizontally along the tangent to the edge of the platform when caught by the kid, find the angular speed of the platform after the event. ($\dfrac{mvR}{I+(M+m)R^2}$)

18. Suppose the platform of the previous problem is brought to rest with the ball in the hand of the kid at the rim. The kid throws the ball to his friend in the horizontal direction tangential to the rim with a velocity v as seen by his friend. Find the angular velocity with which the platform will start rotating. ($\dfrac{mvR}{I+MR^2}$)

19. Suppose the platform with the kid in the previous problem is rotating in anticlockwise direction at an angular speed of ω. The kid starts walking along the rim with a speed v relative to the platform also in the anticlockwise direction. Find the new angular speed of the platform. ($\omega - \dfrac{mvR}{I+MR^2}$)

20. A uniform rod of length L lies on a smooth horizontal table. A particle moving on the table strikes the rod perpendicularly at an end and stops. Find the distance travelled by the centre of the rod by the time it turns through a right angle. Show that if the mass of the rod is 4 times that of the particle, the collision is elastic.



21. A thin uniform triangular sheet of mas m has sides AB = BC = L. What is the moment of inertia about an axis AC lying in the plane of the sheet?

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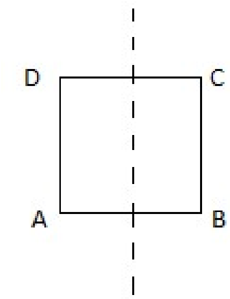
a) $mL^2/12$ b) $mL^2/6$ c) $mL^2/3$ d) $2mL^2/3$

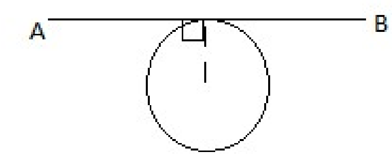


22. Four thin metal rods of mass m and length L each are welded to form a square ABCD as shown. What is the M.I of the composite structure about a line that bisects rods AB and CD and is in the plane of the structure?

a) $mL^2/6$ b) $mL^2/3$ c) $mL^2/2$ d) $2mL^2/3$

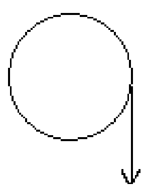
23. Three metal rods each of mass m and length L are welded to form an equilateral triangle. The M.I of the composite structure about an axis passing through the centre of mass and perpendicular to the plane of the structure is

a) $Ml^2/2$ b) $mL^2/4$ c) $mL^2/8$ d) $mL^2/12$



24. A thin wire of length L and uniform linear mass density ρ is bent into a circular loop as shown. The M.L of the loop about the axis AB is

a) $\rho L^3/8\pi^2$ b) $\rho L^3/16\pi^2$ c) $5\rho L^3/16\pi^2$ d) $3\rho L^3/8\pi^2$



25. A hollow cylinder of mass 3 kg and radius 40 cm is free to rotate about a fixed horizontal axis passing through its centre as shown. A rope wound round it is pulled with a force of 30 N. The angular acceleration of the cylinder (in rad/sec^2) is

a) 10   b) 15   c) 20   d) 25

26. What is the linear acceleration (in m/s^2) of a point on the rope?

a) 5   b) 7.5   c) 10   d) 12.5

27. A mass is whirled in a circular path with a constant angular velocity and its angular momentum is L. If the string is now halved keeping the angular velocity the same, the angular momentum is

a) L/4   b) L/2   c) L   d) 2L

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28. A solid sphere A and a hollow sphere B have equal masses and equal outer radii. The densities of A and B are d_A and d_B. If their M.I about their diameters are I_A and I_B respectively, then

a) $I_A = I_B$ b) $I_A > I_B$ c) $I_A < I_B$ d) $I_A/I_B = d_A/d_B$

29. M.I of a uniform horizontal solid cylinder of mass m about an axis passing through its edge and perpendicular to the axis of the cylinder when its length is 6 times its radius R is

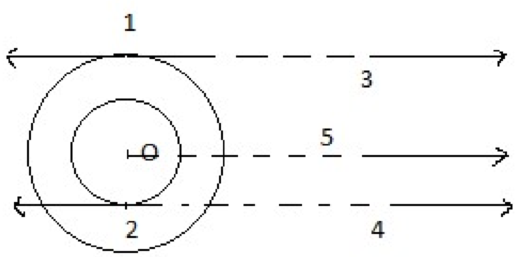
a) $39mR^2/4$ b) $39mR/4$ c) $49mR/4$ d) $49mR^2/4$

30. Two circular loops A and B of radii r_A and r_B respectively are made of uniform wire. The ratio of their M.I about axes passing through their centres and perpendicular to their planes is $I_B/I_A = 8$. Then r_B/r_A is equal to

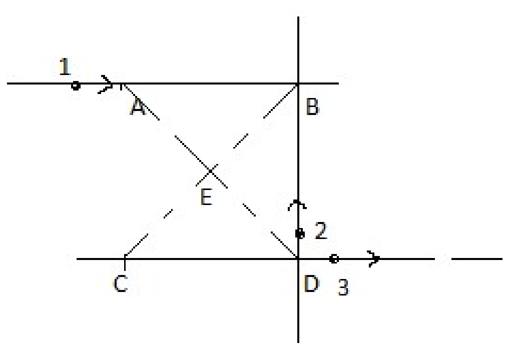
a) 2 b) 4 c) 6 d) 8

31. At a certain time a 0.25 kg object has a position vector $\vec{r} = 2\hat{i} - 2\hat{k}$ in meters. At that instant its velocity in m/s is $\vec{v} = -5\hat{i} + 5\hat{k}$ and the force in newton acting on it is $\vec{F} = 4\hat{j}$. What is the angular momentum of the object about the origin? What torque acts on it?

32. A 2 kg particle like object moves in a plane with velocity components $v_x = 30$ m/s and $v_y = 60$ m/s as it passes through the point with coordinates (3, -4) m. Just then what is its angular momentum relative to the origin and relative to the point (-2, -2) m?

33. 

Particles 1 and 2 move around the point O in circles of radii 2 m and 4 m respectively. Particles 3 and 4 travel in straight lines at perpendicular distances of 4 m and 2 m from the point O. Particle 5 moves directly away from O. All the particles have the same mass and the same constant speed. Rank the particles according to the magnitudes of their angular momentum about O, greatest first. Which particles have negative angular momentum about point O?

34. 

Three particles 1, 2 and 3 are of the same mass and are moving with the same speed indicated by velocity vectors. Points A, B, C and D form a square with E as the centre. Rank the points according to the magnitude of the net

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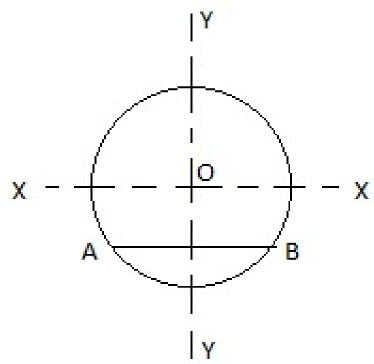
angular momentum of the three particle system when measured about the points. Greatest first.



A uniform rod AB of mass m and length L is attached to a uniform ring of mass 2m and radius L as shown.

35. The M.I of the system about an axis passing through the centre of the ring and perpendicular to the plane of the ring is

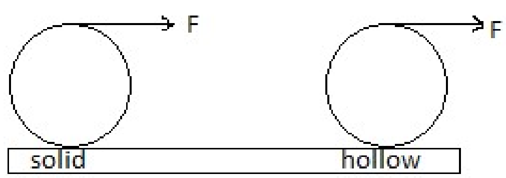
a) $34mL^2/12$ b) $25mL^2/12$ c) $22mL^2/12$ d) $13mL^2/12$

36. The M.I of the system about an axis passing through the centre of mass of the system and perpendicular to the plane of the ring is (in multiple of mL^2)

a) 33/12 b) 31/12 c) 7/3 d) 25/12

37. The M.I of the system about an axis passing through the end A of the rod and perpendicular to the plane of the ring is (in multiple of mL^2)

a) 4/3 b) 7/3 c) 11/3 d) 13/3

38. 

A constant horizontal force F is applied on top of a solid and hollow sphere of the same mass and radius kept on a sufficiently rough horizontal surface. Let a_1 and a_2 be their linear accelerations. Then

a) $a_1 = a_2$ b) $a_1 > a_2$ c) $a_1 < a_2$ d) can't say

39. A solid sphere of mass 2 kg and radius 5 cm rolls without slipping along a horizontal plane. The velocity of its centre of mass is 10 cm/s. The kinetic energy of the sphere is

a) 0.07 J b) 0.014 J c) 0.025 J d) 1.25 J

40. A ring is rolling on a surface without slipping. What is the ratio of its translational to rotational kinetic energies?

a) 5: 7 b) 2: 5 c) 2: 7 d) 1: 1

41. A body rolls down an inclined plane. If its rotational kinetic energy is 40 % of its translational kinetic energy then the body is

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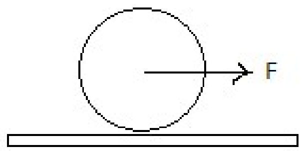
a) Cylinder b) ring c) disc d) solid sphere

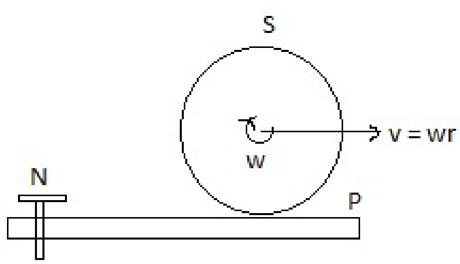
42. A loop and a disc have the same mass and roll without slipping with the same linear velocity v. If the total kinetic energy of the loop is 8 J, the kinetic energy of the disc must be

a) 6 J b) 8 J c) 12 J d) 16 J



43. A solid sphere of mass 2 kg is pulled by a constant horizontal force through its centre on a rough horizontal surface with $\mu = 0.5$. The maximum value of F so that the sphere rolls without slipping is ($g = 10\ ms^{-2}$)

a) 70 N b) 25 N c) 40 N d) 35 N



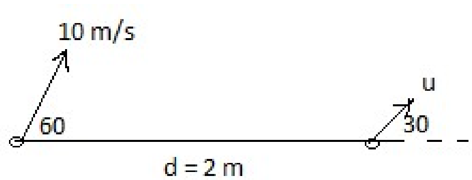
44. A sphere S rolls without slipping, moving with a constant speed on a plank P. There is sufficient friction between the sphere and the plank to prevent slipping while the lower surface of the plank in contact with the ground is smooth. Initially the plank is fixed to the ground by a pin N. If N is suddenly removed,

a) S will begin to slip on P.

b) P will begin to move backwards.

c) The speed of S will decrease and its angular velocity will increase.

d) There will be no change in the motion of S and P will still be at rest.



45. Two particles A and B are situated at a distance of 2 m. A has a velocity of 10 m/s and B has a velocity u m/s. The distance between A and B is constant. The angular velocity of B w.r.t A is

a) $5\sqrt{3}$ m/s b) $5/\sqrt{3}$ m/s c) $10\sqrt{3}$ m/s d) $10/\sqrt{3}$ m/s

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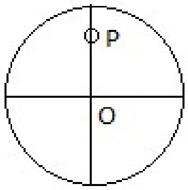
46. 

A disc of radius R rolls on a horizontal ground with linear acceleration a to the right and angular acceleration α clockwise. It has a velocity v at the instant shown and angular velocity ω also clockwise. What is the magnitude of the acceleration of the point P, distance r from the centre?

a) $\sqrt{(a+\alpha r)^2+(r\omega^2)^2}$
b) $\frac{\alpha r}{R}$
c) $\sqrt{\alpha^2 r^2+r^2\omega^4}$
d) αr

47. A solid sphere and a hollow sphere of equal mass and radius are placed over a rough horizontal surface after rotating it about its centre of mass with the same angular velocity. Once the pure rolling starts, let v_1 and v_2 be the velocities of their centre of mass. Then

a) $v_1 = v_2$
b) $v_1 > v_2$
c) $v_1 < v_2$
d) data insufficient

48. In the above problem if t_1 and t_2 be the times after which pure rolling of solid and hollow spheres starts, then

a) $t_1 = t_2$
b) $t_1 > t_2$
c) $t_1 < t_2$
d) none

49. A homogeneous cylinder of mass m and radius r is pulled on a horizontal plane by a horizontal force F acting through its centre of mass. Assuming rolling without slipping the angular acceleration of the cylinder is

a) 3F/2mr
b) 2F/3mr
c) F/2mr
d) 3F/4mr

50. The linear velocity of a particle moving with angular velocity $\vec{\omega}=2\hat{k}$ having position vector $\vec{r}=2\hat{i}+2\hat{j}$ is

a) $4(\hat{i}-\hat{j})$
b) $4(-\hat{i}+\hat{j})$
c) $4\hat{i}$
d) - $4\hat{i}$